



GUM Uncertainty Budget Calculator — Instruction Guide

The GUM Uncertainty Budget Calculator turns a list of uncertainty contributors into u_c , v_{eff} , k and a reported expanded uncertainty U , computed exactly in code.

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1. Purpose and application

The calculator evaluates a linear, uncorrelated uncertainty budget of 1 to 15 contributors by the GUM method. It reports each contributor's share, the combined standard uncertainty u_c , the effective degrees of freedom v_{eff} , the coverage factor k and the expanded uncertainty U .

Use it to:

- build or check a calibration uncertainty budget from certificate values, specifications and repeat readings;
- see which contributor dominates the variance and which limits v_{eff} ;
- compare a laboratory's fixed k (for example 2) with the k that v_{eff} supports;
- teach the full chain from contributor table to reported result.

The card is for education and training. A production budget must follow your laboratory's documented procedure and be reviewed by a qualified metrologist.

2. Standards basis

Standard	What the calculator takes from it
JCGM 100:2008 (GUM) 5.1.2	The law of propagation of uncertainty for uncorrelated inputs: $u_c^2 = \sum c_i^2 u_i^2$
JCGM 100:2008 4.2 and 4.3	Type A evaluation ($u = s/\sqrt{m}$, $v = n - 1$) and Type B evaluation from other information; sources identified per 4.3.1
JCGM 100:2008 G.4 and Table G.2	The Welch-Satterthwaite formula for v_{eff} , truncation of v_{eff} to the next lower integer (G.4.1 permits interpolation or truncation; the calculator truncates, as H.1.6 does), and the Student's t values used as k
JCGM 100:2008 G.4.2 (Equation G.3)	Degrees of freedom of a Type B input from its stated reliability: $v = \frac{1}{2} (R/100)^{-2}$
JCGM 100:2008 7.2.6	Rounding a reported uncertainty up rather than to nearest
UKAS M3003 (Ed. 6, 2024) Appendix C	The teaching note when a rectangular or U-shaped input dominates ($u_N/u_R \leq 0.70$)
NIST SP 811 (2008) Appendix B	The exact conversion factors used inside a unit domain
ISO/IEC 17025:2017 7.11.2	Validation of calculation software before use and after changes, applied in MT-VAL-GB-002

3. How the calculator ensures accurate results

Code computes every number; the language model only explains the finished results. Each typed value is read as an exact fraction, and the budget is computed in squares with rational arithmetic. Displayed roots (u_i , u_c , U) are decided by comparing squares of integers, and v_{eff} is truncated by integer division. Display rules round uncertainties up, so a shown value never claims more than the computed value supports. The model receives the values as an authoritative block and is forbidden to recompute or re-round them.

MT-VAL-GB-002 Rev B validated this calculator at production commit 5cbb347. Over 8,000 random budgets (39,692 contributors) it gave 0 mismatches against an independent Python reference on every displayed and full-precision value. All 28 rows of Table G.2 were exercised, including ∞ . The set held 1,980 budgets with k stated, 407 with $v_{\text{eff}} = \infty$, 4,068 with a unit conversion and 1,146 with a relative input. All 7 edge and rejection cases and all 5 live tests passed. The method is described in the white paper [Deterministic Computation in AI-Assisted Metrology](#) (Racen, 2026).

4. Input fields

The form has budget fields, then one block of contributor fields per contributor, added with the + button (1 to 15). The help text in the last column is the form's own wording.

Budget fields (once per budget)

Field	Required	Format or options	What it means (form help text)
Measurand	Optional	Free text, e.g. length of a 50 mm end gauge	What is being measured. Optional.
Unit domain	Required	23 quantity domains, or Other (same unit on both values) (5.1)	The kind of quantity of the RESULT. Same-quantity contributors are converted into the result unit; a contributor of a different quantity takes its own unit and a sensitivity coefficient. Choose Other only for a unit that is not listed.
Measurand value y	Optional	Bare number, e.g. 10.0000	Optional: the estimate y of the measurand, as a number, in the result unit. Needed for relative inputs (% or ppm of y); with it the card also gives the result statement $y \pm U$ and the relative U .
Result unit	Required	Units of the chosen domain; typed under Other	Unit of the result (not %).

Field	Required	Format or options	What it means (form help text)
Coverage	Required	Confidence level (k from v_{eff}) (default); k stated	Confidence level: k is the Student's t value for the effective degrees of freedom (GUM Table G.2). k stated: a coverage factor your laboratory fixes, for example 2.
Confidence level	Required when Coverage is Confidence level	68.27 %; 90 %; 95 %; 95.45 % (default); 99 %; 99.73 %	The level of confidence of the expanded uncertainty. 95.45 % is the level of $k = 2$ for a normal distribution.
Coverage factor k	Required when Coverage is k stated	Number from 1 to 10, e.g. 2	The coverage factor k, between 1 and 10. The card shows the Welch-Satterthwaite k for comparison.
Source documentation	Optional	Free text, e.g. calibration certificates, datasheets, procedure	The certificates, datasheets and procedures behind the contributors (GUM 4.3.1). Optional.

Contributor fields (one block per contributor)

Field	Required	Format or options	What it means (form help text)
Source	Required	Free text, e.g. gauge block certificate	Name of this contributor.
How it is entered	Required	Four entry forms (4.2)	u: a standard uncertainty you already have (the Type B Estimator gives one from a limit, a resolution or a datasheet). Type A: from the standard deviation s of n readings. Same quantity: in the units of the result. Different quantity: in its own unit, with a sensitivity coefficient c .
Evaluation type	Required for the two u forms	Type A; Type B (default)	Type A: from repeated readings; Type B: from other information.
Distribution it was evaluated from	Optional, Type B only	Not stated or normal (default); Rectangular; Triangular; U-shaped	Optional. Only used for a teaching note when a rectangular or U-shaped input dominates; it never changes the arithmetic.

Field	Required	Format or options	What it means (form help text)
Value of u	Required, u same quantity	Bare number, e.g. 0.0004	A bare number, in the unit beside it, or % or ppm of y.
Unit of u	Required, u same quantity	Units of the result's domain, plus % of y and ppm of y	Unit of u, or % of y, ppm of y.
Value of u (other quantity)	Required, u different quantity	Bare number, e.g. 0.58e-6	A bare number, in the unit you type beside it.
Unit of u (other quantity)	Required, u different quantity	Typed text	The unit of this input, typed.
Value of s	Required, Type A same quantity	Bare number, e.g. 0.012	The experimental standard deviation s of the readings, a bare number.
Unit of s	Required, Type A same quantity	Units of the result's domain (no % of y)	Unit of s.
Value of s (other quantity)	Required, Type A different quantity	Bare number, e.g. 0.012	The experimental standard deviation s of the readings, a bare number.
Unit of s (other quantity)	Required, Type A different quantity	Typed text	The unit of this input, typed.
Number of readings n	Required, both Type A forms	Whole number, at least 2, e.g. 10	Readings that gave s ($v = n - 1$).
Readings averaged m (blank = n)	Optional, both Type A forms	Whole number, at least 1, e.g. 5	Readings averaged ($u = s/\sqrt{m}$). Blank = n.
Sensitivity coefficient c (result unit per this unit)	Required, both different-quantity forms	Signed number, e.g. -0.5	Signed. Result unit per unit of this input; 0 is allowed.
Degrees of freedom (Type A)	Required when Evaluation type is Type A	Number greater than 0, e.g. 9	A Type A u needs its degrees of freedom v ($n - 1$ for n readings).

Field	Required	Format or options	What it means (form help text)
How the degrees of freedom are known	Required when Evaluation type is Type B	Infinite (not stated) (default); Enter v; From reliability	Infinite: the bounds are treated as exactly known (GUM G.6.4). Reliability: $v = \frac{1}{2} (R/100)^{-2}$ (GUM G.4.2).
Degrees of freedom v	Required when Enter v is chosen	Number greater than 0, e.g. 50	Greater than zero; not necessarily a whole number.
Reliability of the estimate (%)	Required when From reliability is chosen	Greater than 0, at most 100, e.g. 25	How reliable the estimate is, % greater than 0 and at most 100 (25 gives $v = 8$).

Every number is entered bare, with a period as the decimal mark. Scientific notation such as 0.58e-6 is accepted.

4.1 Coverage

- **Confidence level (k from v_eff).** The card computes v_eff and reads k from GUM Table G.2 at the chosen level. Confidence level then appears with six options.
- **k stated.** The card uses your k as entered. Coverage factor k then appears. The card still computes v_eff and shows the Welch-Satterthwaite k at 95.45 % for comparison.

4.2 How it is entered

Option	Fields shown	How u_i is formed
Standard uncertainty u (same quantity as the result)	Evaluation type, Value of u, Unit of u	u as entered, converted into the result unit; $c = +1$
Standard uncertainty u (different quantity)	Evaluation type, Value of u (other quantity), Unit of u (other quantity), Sensitivity coefficient c	u in its own unit, multiplied by c
Type A from s and n (same quantity as the result)	Value of s, Unit of s, Number of readings n, Readings averaged m	$u = s/\sqrt{m}$, converted into the result unit; $v = n - 1$
Type A from s and n (different quantity)	Value of s (other quantity), Unit of s (other quantity), n, m, Sensitivity coefficient c	$u = s/\sqrt{m}$ in its own unit, multiplied by c; $v = n - 1$

The two Type A forms take their degrees of freedom from n, so they show no evaluation-type or degrees-of-freedom fields.

4.3 Evaluation type and degrees of freedom

- **Type A** (u forms only) shows Degrees of freedom (Type A). Enter v for the readings behind u.
- **Type B** shows How the degrees of freedom are known and Distribution it was evaluated from.

- **Infinite (not stated)** treats the input as exactly known. The card records this choice in a note; it is never assumed silently.
- **Enter v** shows Degrees of freedom v. Use it when a certificate states v for its uncertainty.
- **From reliability** shows Reliability of the estimate (%). A reliability of 25 % gives $v = \frac{1}{2} \times 0.25^{-2} = 8$.

4.4 Distribution it was evaluated from

This choice never changes the arithmetic: enter u already divided by its divisor. Rectangular or U-shaped selections enable one teaching note when that input dominates u_c (section 7). Not stated or normal and Triangular never trigger it.

5. Units and unit conversion

The Unit domain fixes the kind of quantity of the result. Every same-quantity contributor is brought into the result unit; a different-quantity contributor keeps its typed unit and is carried by c.

5.1 Domains and units offered

Domain	Units offered for the result, u and s
Voltage	MV, kV, V, mV, μ V, nV, pV
Current	kA, A, mA, μ A, nA, pA
Resistance	T Ω , G Ω , M Ω , k Ω , Ω , m Ω , $\mu\Omega$
Capacitance	F, mF, μ F, nF, pF
Inductance	H, mH, μ H, nH
Conductance	S, mS, μ S, nS
Electric charge	C, mC, μ C, nC, pC
Pressure	Pa, mPa, μ Pa, kPa, MPa, GPa, bar, mbar, psi, atm, Torr, mTorr, mmHg (conventional), inHg (60 °F), inHg (32 °F)
Temperature	K, mK, °C, °F, °R (as temperature differences)
Torque	N·m, N·cm, mN·m, μ N·m, kN·m, lbf·ft, lbf·in, ozf·in
Force	N, mN, μ N, kN, MN, dyn, lbf, ozf, kgf
Mass	kg, g, mg, μ g, ng, t, lb, oz, gr, slug
Length	m, cm, mm, μ m, nm, pm, km, in, ft, yd, mi, mil (0.001 in), nmi
Time	s, ms, μ s, ns, ps, min, h, d
Frequency	Hz, mHz, kHz, MHz, GHz

Domain	Units offered for the result, u and s
Angle	rad, °, mrad, ', ", gon
Rotational speed	r/min, r/s, rad/s
Area	m ² , cm ² , mm ² , in ² , ft ² , yd ² , acre, ha
Volume	L, mL, µL, m ³ , cm ³ , gal, liq qt, liq pt, fl oz, in ³ , ft ³
Energy	J, mJ, µJ, kJ, MJ, cal_IT, cal_th, kcal_IT, kcal_th, W·h, kW·h, Btu_IT, ft·lbf
Power	W, mW, µW, nW, pW, kW, MW, hp, hp (electric), hp (metric), Btu_IT/h
Magnetic flux density	T, mT, µT, nT, G
Magnetic flux	Wb, mWb, µWb, Mx
Other (same unit on both values)	No list: the unit is typed

The form shows each unit with its name, for example "microvolt (µV)". Unit of u also offers % of y and ppm of y in every listed domain.

5.2 How a conversion is made

- **Exact factors.** Each factor is built from its definition in rational arithmetic: 1 in = 0.0254 m, 1 lb = 0.453 592 37 kg, $g_n = 9.806\,65\text{ m/s}^2$, 1 atm = 101 325 Pa, and so on (NIST SP 811 Appendix B). The value is multiplied by the ratio of the two factors before squaring, so no rounding enters the budget.
- **Temperature is an interval.** An uncertainty is a temperature difference: 1 °C = 1 K and 1 °F = 1 °R = 5/9 K. No offset is applied.
- **Tabulated factors.** The two inHg units use SP 811's tabulated values (3 376.85 Pa and 3 386.38 Pa), not definitions; the conversion note says "tabulated".
- **π families are not mixed.** Degree, arcminute, arcsecond and gon differ from radian and milliradian by π; r/min and r/s differ from rad/s by π. A contributor in one family against a result in the other is refused, because the conversion would not be exact.
- **The note.** Each conversion is listed on the card, e.g. "Contributor 1: u 4 µV converted to 0.000004 V (×0.000001, exact)". A value that must be rounded for that note is rounded up at the entered significant figures + 3, at most 7. The budget itself uses the exact value.

5.3 Relative inputs, different quantities and Other

- **% of y and ppm of y.** $u = p/100 \times |y|$ or $p \times 10^{-6} \times |y|$, in the result unit. Measurand value y must be entered and non-zero. Relative units are offered only for a same-quantity standard uncertainty u, not for s.
- **Different quantity.** The unit is free text (for example °C, 1/°C, %RH) and is never converted. Enter c in result unit per that typed unit, so $c \times u$ lands in the result unit. Relative units are refused here.

- **Other.** Every unit is typed, and every same-quantity value must use exactly the result unit text. Nothing is converted, and the card says so.

6. Calculation method

The budget is computed in squares, exactly, in five steps. Only the displayed values are rounded, and only at the end.

Step 1. Each contribution. For a u form, u_i is the entered value converted into the result unit (or taken relative to $|y|$). For a Type A form, $u_i^2 = s^2/m$. The contribution is its square times c_i^2 ($c_i = 1$ for the same quantity):

$$w_i = c_i^2 u_i^2, \quad u_i^2 = \frac{s_i^2}{m_i} \text{ (Type A)}, \quad \nu_i = n_i - 1 \text{ (Type A from } s, n)$$

Step 2. Combined standard uncertainty by the law of propagation for uncorrelated inputs (GUM 5.1.2):

$$u_c = \sqrt{\sum_{i=1}^N c_i^2 u_i^2}$$

Step 3. Effective degrees of freedom by the Welch-Satterthwaite formula (GUM G.4.1). Only contributors with finite ν_i enter the denominator. If every ν_i is infinite, $\nu_{\text{eff}} = \infty$ exactly.

$$\nu_{\text{eff}} = \frac{u_c^4}{\sum_{i=1}^N \frac{c_i^4 u_i^4}{\nu_i}}$$

A Type B ν from reliability R (in %) is $\nu = \frac{1}{2} (R/100)^{-2}$ (GUM G.4.2). ν_{eff} is truncated to the next lower integer. GUM G.4.1 permits interpolation or truncation; the calculator truncates, as H.1.6 does. 16.76 becomes 16, never 17.

Step 4. Coverage factor. With Confidence level, k is read from Table G.2 at the truncated ν_{eff} and the chosen level:

- ν_{eff} on a tabulated row uses that row (rows 1 to 20, then 25, 30, 35, 40, 45, 50, 100);
- ν_{eff} between rows uses the next lower row ($\nu_{\text{eff}} = 22$ uses row 20), which gives the larger k ;
- ν_{eff} below 2 uses row 1; below 1 the card adds the note " ν_{eff} below 1";
- finite ν_{eff} above 100 uses row 100 ($k = 2.025$ at 95.45 %), not the ∞ row;
- $\nu_{\text{eff}} = \infty$ uses the ∞ row ($k = 2.000$ at 95.45 %, 1.960 at 95 %).

The table is never interpolated. With k stated, your k is used and the card adds the Welch-Satterthwaite k at 95.45 % for comparison.

Step 5. Expanded uncertainty.

$$U = k u_c, \quad U^2 = k^2 u_c^2 \text{ (decided exactly)}$$

6.1 Display and rounding rules

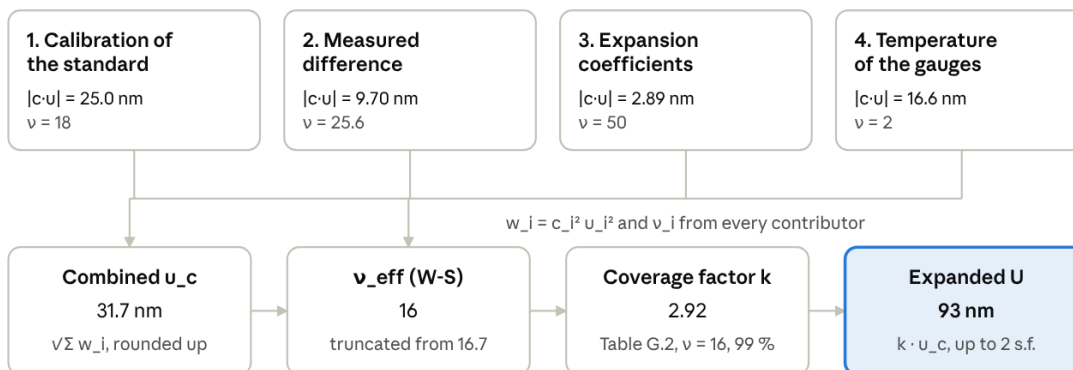
Value	Rule on the card	Why
U	Rounded up to 2 significant figures	A rounded-up U is never smaller than the computed one; GUM 7.2.6 permits rounding up
$u_c, u_i, c \cdot u $	Rounded up to 3 significant figures	Same reason, one guard digit

Value	Rule on the card	Why
u_c and U, full precision	10 significant figures, nearest (ties to even)	For checking and for carrying into another budget
v_eff	Integer, truncated; raw value truncated to 1 decimal, e.g. "16 (truncated from 16.6)"	GUM G.4.1 permits interpolation or truncation; the calculator truncates, as H.1.6 does. The raw value never reads above the next integer
k from Table G.2	As printed: 2 decimals, 3 decimals on the 100 and ∞ rows	Matches the published table
y in the result statement	Rounded to the last digit of U	GUM 7.2.6
Relative U	Rounded up to 2 significant figures, in % (or ppm below 0.1 %) of y	Same as U
Shares of variance and of the v_eff denominator	Nearest, 1 decimal	Shares are not uncertainties

A rounded value keeps its significant zeros (25.0 nm, 0.0000132 V).

Four contributors set u_c and v_eff; k from Table G.2 turns u_c into U

Example 1, GUM H.1 gauge block at 99 %, values as the card shows them



budget flow · 4 contributors to U

Figure 1. The budget flow for Example 1: each contributor's c·u and v feed u_c and v_eff; the truncated v_eff picks k from Table G.2; $U = k \cdot u_c$.

7. Reading the results

The headline states U with its basis. With Confidence level it reads "U = 93 nm (k = 2.92, $v_{\text{eff}} = 16$, 99 %)". With k stated it reads "U = 0.0045 V (k = 2 stated, $v_{\text{eff}} = 267$)".

Card row	What it shows
Combined Standard Uncertainty	u_c , rounded up to 3 s.f., in the result unit
Combined Standard Uncertainty Full Precision	u_c to 10 s.f.
Effective Degrees Of Freedom	Truncated v_{eff} and its one-decimal value, or " ∞ (every degree of freedom is infinite)"
Coverage Factor	k as used
Coverage Basis	The Table G.2 row and level used, or "k stated by the user"
Confidence Level	The level chosen (Confidence level mode only)
Expanded Uncertainty	U, rounded up to 2 s.f.
Expanded Uncertainty Full Precision	U to 10 s.f.
Result Statement	The GUM's full result sentence; with y it starts "y = (y $\pm$ U) unit"
Relative Expanded Uncertainty	U as % or ppm of y , when y is entered
Dominant Contributor By Variance	The contributor(s) with the largest $c^2 u^2$ and their share
Dominant Contributor By Degrees Of Freedom	The contributor(s) with the largest share of the v_{eff} denominator (only when some v is finite)
Budget — n. Source	One row per contributor: form, value as entered, u, c, c·u , v, share of variance, share of the v_{eff} denominator
Unit Conversions — n. Source	The conversion note for each converted contributor
Unit	The result unit

Below the rows the card lists notes. Recorded facts include "Degrees of freedom defaulted to infinity", "Degrees of freedom from reliability", "Type A from s, n and m", "Relative input" and "Unit converted". "Source documentation" either names the sources or asks for them (GUM 4.3.1).

Teaching notes never change a number:

- **Model scope:** always shown; correlation and nonlinearity are not treated.
- **One contributor dominates the variance:** one w_i is more than half of u_c^2 .

- **A small- v contributor limits v_{eff} :** one contributor carries more than half of the v_{eff} denominator.
- **Stated k differs from the Welch-Satterthwaite k :** k stated and the 95.45 % Table G.2 k differ.
- **v_{eff} below 1:** row 1 was used.
- **Zero sensitivity coefficient:** some $c = 0$.
- **Dominant rectangular or U-shaped contributor:** the other contributions satisfy $u_N/u_R \leq 0.70$ (UKAS M3003 Appendix C). k is not changed.
- **Measurand value has fewer decimals than U :** y was padded to U 's last digit.

The PDF, Word, spreadsheet and chat exports carry the same strings as the card, from one presentation function.

8. Worked examples

Every result below was produced by the production calculator from the form text shown, and checked by hand in exact arithmetic.

Example 1: GUM H.1 gauge block (published known answer)

JCGM 100:2008 H.1 reports $U = 93$ nm with $k = 2.92$ and $v_{\text{eff}} = 16$ at 99 %. The MT-VAL-GB-001 harness carries this budget as a known answer. Here it is entered with the GUM's sensitivity coefficients.

Field	Entered
Measurand	length of a 50 mm end gauge
Unit domain	Length
Result unit	nanometer (nm)
Coverage	Confidence level (k from v_{eff})
Confidence level	99 %
Source documentation	JCGM 100:2008 H.1

Source	How it is entered	Evaluation type	Value, unit	c	Degrees of freedom
Calibration of the standard	Standard uncertainty u (same quantity as the result)	Type B	25, nanometer (nm)	(none)	Enter v : 18
Measured difference	Standard uncertainty u (same quantity as the result)	Type B	9.7, nanometer (nm)	(none)	Enter v : 25.6

Source	How it is entered	Evaluation type	Value, unit	c	Degrees of freedom
Difference in expansion coefficients	Standard uncertainty u (different quantity)	Type B	0.577e-6, 1/°C	5e6	Enter v: 50
Difference in temperature of the gauges	Standard uncertainty u (different quantity)	Type B	0.028868, °C	-575	Enter v: 2

GUM H.1.3.5, H.1.3.6 and Table H.1 print $u(\delta\alpha) = 0.58 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ and $u(\delta\theta) = 0.029 \text{ }^{\circ}\text{C}$. Their unrounded values, entered here, are $1 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}/\sqrt{3} = 0.577 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ and $0.05 \text{ }^{\circ}\text{C}/\sqrt{3} = 0.028868 \text{ }^{\circ}\text{C}$. Entering the printed 0.58e-6 and 0.029 gives $u_c = 31.8 \text{ nm}$ and a 16.7 nm temperature term; U, k and v_{eff} are unchanged.

With $l_s = 50 \text{ mm}$ and $\bar{\theta} = -0.1 \text{ }^{\circ}\text{C}$, c for the expansion term is $-l_s\bar{\theta} = +5 \times 10^6 \text{ nm}\cdot^{\circ}\text{C}$. c for the temperature term is $-l_s\alpha_s = -575 \text{ nm}/^{\circ}\text{C}$.

Measured difference is not a pure Type A input: in GUM H.1 it combines two Type A parts and one Type B part, and its $v = 25.6$ is itself a Welch-Satterthwaite value. It is therefore entered as Type B with Enter v, the form's route for a stated v.

Hand calculation.

- Squares: $25^2 = 625$; $9.7^2 = 94.09$; $(0.577 \times 10^{-6} \times 5 \times 10^6)^2 = 2.885^2 = 8.323225$; $(0.028868 \times 575)^2 = 16.5991^2 = 275.53012081 \text{ nm}^2$.
- $u_c^2 = 1002.94334581 \text{ nm}^2$, so $u_c = 31.669281... \text{ nm}$, shown rounded up as 31.7 nm.
- Denominator: $625^2/18 + 94.09^2/25.6 + 8.323225^2/50 + 275.53012081^2/2 = 21701.39 + 345.82 + 1.39 + 37958.42 = 60007.02$.
- $v_{\text{eff}} = 1002.94334581^2 / 60007.02 = 16.763...$, truncated to 16.
- Table G.2, $v = 16$, 99 %: $k = 2.92$.
- $U = 2.92 \times 31.669281 = 92.474300... \text{ nm}$, rounded up to 93 nm.

Calculator result rows.

Card row	Value
Headline	$U = 93 \text{ nm}$ ($k = 2.92$, $v_{\text{eff}} = 16$, 99 %)
Combined Standard Uncertainty	31.7 nm
Combined Standard Uncertainty Full Precision	31.66928079 nm
Effective Degrees Of Freedom	16 (truncated from 16.7)
Coverage Basis	GUM Table G.2, $p = 99 \%$, $v = 16$ row
Expanded Uncertainty	93 nm

Card row	Value
Expanded Uncertainty Full Precision	92.47429991 nm
Dominant Contributor By Variance	1. Calibration of the standard (62.3 % of the variance)
Dominant Contributor By Degrees Of Freedom	4. Difference in temperature of the gauges (63.3 % of the v_{eff} denominator)
Budget — 3. Difference in expansion coefficients	Type B u (v entered); $0.577\text{e-}6$ $1/^{\circ}\text{C}$; $u = 0.000000577$ $1/^{\circ}\text{C}$; $c = +5\text{e}6$ nm per $1/^{\circ}\text{C}$; $ c \cdot u = 2.89$ nm; $v = 50$; 0.8 % of the variance; 0.0 % of the v_{eff} denominator
Budget — 4. Difference in temperature of the gauges	Type B u (v entered); 0.028868 $^{\circ}\text{C}$; $u = 0.0289$ $^{\circ}\text{C}$; $c = -575$ nm per $^{\circ}\text{C}$; $ c \cdot u = 16.6$ nm; $v = 2$; 27.5 % of the variance; 63.3 % of the v_{eff} denominator

Interpretation. The calculator reproduces the GUM's budget: the temperature term is 16.6 nm, u_c is 31.7 nm, v_{eff} is 16.7 truncated to 16, $k = 2.92$ and $U = 93$ nm. The GUM prints 93 nm by multiplying a rounded u_c of 32 nm by 2.92; the calculator reaches 93 nm by rounding the exact 92.47 nm up. The temperature term has only 2 degrees of freedom, yet it holds 63.3 % of the v_{eff} denominator. It pulls v_{eff} down to 16 and raises k from 2.576 (the ∞ row at 99 %) to 2.92.

Example 2: 10 V source with conversions and a relative input

This budget uses all four features a working budget usually needs: a unit conversion, a relative specification, Type A readings and a temperature coefficient.

Field	Entered
Measurand	10 V output of a DC voltage source
Unit domain	Voltage
Measurand value y	10.0000
Result unit	volt (V)
Coverage	Confidence level (k from v_{eff})
Confidence level	95.45 %
Source documentation	reference standard certificate, DMM datasheet, procedure

Source	How it is entered	Details
Reference standard certificate	Standard uncertainty u (same quantity as the result)	Type B; Value of u 4; Unit of u microvolt (μV); Infinite (not stated)
DMM 1-year specification	Standard uncertainty u (same quantity as the result)	Type B; Distribution Rectangular; Value of u 1.155; Unit of u ppm of y; From reliability, 25
Repeatability	Type A from s and n (same quantity as the result)	Value of s 0.015; Unit of s millivolt (mV); n 10; m blank
Ambient temperature	Standard uncertainty u (different quantity)	Type B; Value of u (other quantity) 0.6; Unit of u (other quantity) $^{\circ}\text{C}$; c 0.000002; Infinite (not stated)

Hand calculation (in μV for readability).

- Reference: $4 \mu\text{V} \times 10^{-6} \text{ V}/\mu\text{V} = 0.000004 \text{ V}$ (exact); $w = 16 \mu\text{V}^2$, $v = \infty$.
- DMM: $1.155 \times 10^{-6} \times 10 \text{ V} = 11.55 \mu\text{V}$; $w = 133.4025 \mu\text{V}^2$; $v = \frac{1}{2} \times 0.25^{-2} = 8$.
- Repeatability: $s = 0.015 \text{ mV} = 15 \mu\text{V}$; $u^2 = 15^2/10 = 22.5 \mu\text{V}^2$ ($u = 4.743 \mu\text{V}$); $v = 9$.
- Temperature: $0.6 \times 2 \mu\text{V}/^{\circ}\text{C} = 1.2 \mu\text{V}$; $w = 1.44 \mu\text{V}^2$, $v = \infty$.
- $u_{\text{c}}^2 = 173.3425 \mu\text{V}^2$; $u_{\text{c}} = 13.165960 \mu\text{V}$, shown rounded up as 0.0000132 V.
- $v_{\text{eff}} = 173.3425^2 / (133.4025^2/8 + 22.5^2/9) = 30047.62 / 2280.78 = 13.17$, truncated to 13.
- Table G.2, $v = 13$, 95.45 %: $k = 2.21$. $U = 2.21 \times 13.165960 = 29.0968 \mu\text{V}$, rounded up to $30 \mu\text{V} = 0.000030 \text{ V}$.

Calculator result rows.

Card row	Value
Headline	$U = 0.000030 \text{ V}$ ($k = 2.21$, $v_{\text{eff}} = 13$, 95.45 %)
Combined Standard Uncertainty	0.0000132 V
Effective Degrees Of Freedom	13 (truncated from 13.1)
Coverage Basis	GUM Table G.2, $p = 95.45 \%$, $v = 13$ row
Expanded Uncertainty Full Precision	0.00002909677137 V
Result Statement	$y = (10.000000 \pm 0.000030) \text{ V}$, where the number following the symbol $\pm$ is the numerical value of an expanded uncertainty $U = k \cdot u_{\text{c}}$, with U determined from a combined standard uncertainty $u_{\text{c}} = 0.0000132 \text{ V}$ and a coverage factor $k = 2.21$ based on the t-distribution for $v_{\text{eff}} = 13$ degrees of freedom and defines an interval estimated to have a level of confidence of 95.45 %.

Card row	Value
Relative Expanded Uncertainty	3.0 ppm of y
Dominant Contributor By Variance	2. DMM 1-year specification (77.0 % of the variance)
Unit Conversions — 1. Reference standard certificate	Contributor 1: u 4 μ V converted to 0.000004 V ($\times 0.000001$, exact)
Unit Conversions — 3. Repeatability	Contributor 3: s 0.015 mV converted to 0.000015 V ($\times 0.001$, exact)

Interpretation. The DMM specification is 77.0 % of the variance and 97.5 % of the v_{eff} denominator, so it sets both u_c and k . Lowering the Reliability of the estimate value from 25 % (a better-known specification; 10 % gives $v = 50$) would raise v_{eff} and lower k toward 2. The card also notes that this rectangular input dominates ($u_N/u_R = 0.55$), so a normal-based k is conservative. It adds that y was entered with 4 decimals while U needs 6; the trailing zeros in 10.000000 are placeholders.

Example 3: k stated, with the Welch-Satterthwaite k for comparison

This is live test G3 in MT-VAL-GB-002. Unit domain Voltage, result unit volt (V), Coverage k stated, Coverage factor k_2 .

Source	How it is entered	Details
reference	Standard uncertainty u (same quantity as the result)	Type B; 0.002, volt (V); Infinite (not stated)
temperature	Standard uncertainty u (different quantity)	Type B; 0.5, K; c 0.0001; Infinite (not stated)
repeatability	Type A from s and n (same quantity as the result)	s 0.003, volt (V); n 10

Hand calculation. $w = 4 \times 10^{-6} + (0.5 \times 0.0001)^2 + 0.003^2/10 = 4 \times 10^{-6} + 2.5 \times 10^{-9} + 9 \times 10^{-7} = 4.9025 \times 10^{-6} \text{ V}^2$. $u_c = 0.0022141590 \text{ V}$, shown as 0.00222 V. Only repeatability has finite v (9): $v_{\text{eff}} = (4.9025 \times 10^{-6})^2 / ((9 \times 10^{-7})^2/9) = 267.05$, truncated to 267. $U = 2 \times 0.0022141590 = 0.0044283 \text{ V}$, rounded up to 0.0045 V. Row 100 at 95.45 % gives the comparison $k = 2.025$.

Calculator result rows. Headline: $U = 0.0045 \text{ V}$ ($k = 2$ stated, $v_{\text{eff}} = 267$). Combined Standard Uncertainty: 0.00222 V. Effective Degrees Of Freedom: 267 (truncated from 267.0). Coverage Basis: k stated by the user. Expanded Uncertainty Full Precision: 0.004428317965 V. Note: " k stated = 2; the Welch-Satterthwaite k at 95.45 % for $v_{\text{eff}} = 267$ is 2.025."

Interpretation. $k = 2$ is used as entered. The note shows that $v_{\text{eff}} = 267$ supports $k = 2.025$ for 95.45 %, not 2.000, because finite v_{eff} above 100 uses row 100. Here $U = 0.0045 \text{ V}$ either way ($2.025 \times 0.0022142 = 0.0044837 \text{ V}$ also rounds up to 0.0045 V).

Example 4: every v infinite

Live test G1: four contributors, each Standard uncertainty u (same quantity as the result), Type B, 1 volt (V), Infinite (not stated); Confidence level 95 %. Hand: $u_c = \sqrt{4} = 2$ V; no finite v , so $v_{\text{eff}} = \infty$ and $k = 1.960$; $U = 3.92$ V, rounded up to 4.0 V. The card shows "U = 4.0 V ($k = 1.960$, $v_{\text{eff}} = \infty$, 95 %)", Effective Degrees Of Freedom " ∞ (every degree of freedom is infinite)" and Expanded Uncertainty Full Precision 3.92 V.

9. Refusals and troubleshooting

Input that cannot be read unambiguously is refused with a card that names the field and the fix. Nothing is guessed or defaulted. Each message below was produced by the production calculator.

Message or situation	Cause	What to do
Cannot compute the budget — missing Contributor	No contributor block was entered	Use the + button to add at least one contributor
Cannot compute the budget — invalid Contributor	16 contributors were entered (at most 15), or the numbering has a gap such as 1 then 3	Combine related inputs or enter a sub-budget's result as one contributor; re-enter contributors in the form so they run 1 to N
Cannot compute the budget — invalid Contributor 1: Number of readings n	$n = 1$; n must be a whole number of at least 2	Enter the number of readings that gave s
Cannot compute the budget — invalid Contributor 1: Readings averaged m (blank = n)	$m = 2.5$; m must be a whole number of at least 1	Enter a whole number, or leave it blank for $m = n$
Cannot compute the budget — missing Measurand value y	A contributor is in % of y or ppm of y and y is blank	Enter a non-zero Measurand value y
Cannot compute the budget — the combined standard uncertainty is 0	Every contribution is zero, for example $c = 0$ on the only contributor	Check the contributor values and sensitivity coefficients
Cannot compute the budget — invalid Contributor 1: Value of u ("uses a comma")	0,01 was typed; the decimal mark must be a period	Enter 0.01
Cannot compute the budget — unit mismatch in Contributor 1: Value of u	A unit was typed in the value field, e.g. 0.01 V	Enter the bare number and choose the unit in Unit of u
Cannot compute the budget — unit mismatch in Contributor 1: Unit of u ("different π families")	u in rad against a result in $^\circ$	Enter every quantity in one family: all degrees, minutes and seconds, or all radians

Message or situation	Cause	What to do
Cannot compute the budget — unit mismatch in Contributor 1: Unit of u	% against a result unit of %RH (Other domain)	Use exactly the result unit text, or choose a listed domain
Cannot compute the budget — missing Contributor 1: Degrees of freedom (Type A)	Evaluation type Type A with no v	Enter v, n – 1 for n readings
Cannot compute the budget — invalid Contributor 1: Reliability of the estimate (%)	150 was entered; the limit is greater than 0 and at most 100	Enter the reliability as a percentage, e.g. 25
Cannot compute the budget — invalid Coverage factor k	k = 12; k must be between 1 and 10	Enter your laboratory's k, e.g. 2
Cannot compute the budget — invalid Confidence level	95.5 % is not an option	Choose 68.27 %, 90 %, 95 %, 95.45 %, 99 % or 99.73 %

If U looks too large, read the two dominant-contributor rows first. A contributor with v = 2 to 5 and a large variance share raises k sharply (Example 1). If U looks too small, check that each u was entered as a standard uncertainty: a certificate's expanded U must be divided by its k first.

10. Scope and limitations

- **Linear and uncorrelated only.** Correlated inputs (for example the same standard used twice) and nonlinear models are not offered and were not validated. Use GUM 5.2 or a Monte Carlo evaluation (JCGM 101) for those.
- **You supply c.** The calculator does not derive sensitivity coefficients from a model equation. Enter $c_i = \partial f / \partial x_i$ in result unit per input unit.
- **You supply u.** Divisors for rectangular, triangular or U-shaped limits are not applied here. Use the Type B Estimator first and enter its u.
- **Table G.2 is not interpolated.** The next lower row is always used, which gives a k equal to or larger than an interpolated one. The v = 35, 90 % cell uses the printed 1.70, as the published table does.
- **A dominant rectangular input does not change k.** The card notes it; the k from Table G.2 is kept.
- **Different-quantity units are not checked.** A typed unit and its c are taken as entered.
- **15 contributors at most.** Larger budgets go in as sub-budgets: enter a sub-budget's u_c with its v_{eff} using Enter v.
- **The explanation is model-written.** The numbers on the card are validated; the explanatory text is constrained by its prompt and checked in live tests, but it is outside the numerical validation.

11. References

1. JCGM 100:2008, *Evaluation of measurement data — Guide to the expression of uncertainty in measurement* (GUM): 4.2, 4.3, 5.1.2, 7.2.6, Annex G (G.4, Table G.2), H.1.

2. JCGM 101:2008, *Propagation of distributions using a Monte Carlo method*.
3. UKAS M3003, *The Expression of Uncertainty and Confidence in Measurement*, Edition 6 (2024), Appendix C.
4. NIST SP 811 (2008), *Guide for the Use of the International System of Units*, Appendix B.
5. ISO/IEC 17025:2017, *General requirements for the competence of testing and calibration laboratories*, 7.11.2.
6. AI Metrologist, LLC, MT-VAL-GB-002 Rev B, *Validation report — GUM Uncertainty Budget Calculator*, 18 October 2026.
7. AI Metrologist, LLC, MT-VAL-SUM-001 Rev B, *MetTutor Deterministic Calculators — Validation Summary*, 18 October 2026.
8. Racen, K. (2026). [Deterministic Computation in AI-Assisted Metrology](#).

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